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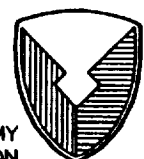
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GEARS (LOCAL SYNTHESIS)



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LOCAL SYNTHESIS AND TOOTH CONTACT ANALYSIS OF FACED-MILLED SPIRAL BEVEL GEARS

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ABSTRACT

A new approach is proposed for the local synthesis of spiral bevel gears. The approach provides contact at the mean contact point with the desired deviation of the transmission error function by a predesigned parabolic function. The orientation of the contact path on the gear tooth surface and the length of the major axis of the instantaneous contact ellipse are also included in the analysis. A tooth contact analysis (TCA) computer program was developed to simulate meshing and contact of the gear tooth surfaces. A numerical example of the process is given.

INTRODUCTION

Computerized simulation of meshing and bearing contact for spiral bevel and hypoid gears [1,2] is a significant achievement that could substantially improve gear technology and quality. (Meshing as used in this paper refers to the location of the center of contact, and bearing contact refers to how the point contact spreads to an elliptical region owing to the load that is applied.) This paper proposes a new approach to the synthesis of face-milled spiral bevel gears and their tooth contact analysis. This new approach is based on the following ideas proposed in [3]: (1) application of the principle of local synthesis, which provides optimal meshing and contact conditions at the mean contact point M and in the neighborhood of M , and (2) application of relations between principal directions and curvatures for surfaces in line or point contact.

The developed local gear synthesis provides (1) the required gear ratio at M ; (2) a localized bearing contact with the desired direction of the tangent to the contact path on the gear tooth surface and the desired length of the major axis of the contact ellipse at M ; and (3) a predesigned parabolic function with a controlled level of transmission errors (6 to 8 arc sec). Such a function of transmission errors enables linear functions of transmission errors caused by misalignment [4] to be absorbed and reduces the vibration level. It is assumed that the duplex method is used to generate the spiral bevel gear (i.e., both sides of the gear teeth are generated simulta-

neously) and that each side of the spiral bevel pinion tooth surface is generated separately.

The new approach does not require either head-cutter tilt or modified roll for gear surface generation. Improved gear meshing and contact can be achieved without tilt or roll. A computer program was developed for determining basic machine-tool settings and tooth contact analysis for the designed gears [5]. The new approach is illustrated with a numerical example.

LOCAL SYNTHESIS

The new approach to local synthesis is based on the following considerations:

(1) Two right-handed trihedrons $S_a(e_f, e_h, n)$ and $S_b(e_s, e_q, n)$ are introduced (Fig. 1). The common origin of the trihedrons coincides with the mean contact point M . The n -axis represents the direction of the common unit normal to the contacting surfaces at M . Unit vectors e_s and e_q are the principal directions of Σ_2 , unit vectors e_f and e_h are the principal directions of Σ_1 , and $\sigma^{(12)}$ is the angle formed between e_f and e_s .

(2) In [4] three linear equations of the following structure were derived:

$$a_{i1}v_s^{(1)} + a_{i2}v_q^{(1)} = a_{i3} \quad (i = 1, 2, 3) \quad (1)$$

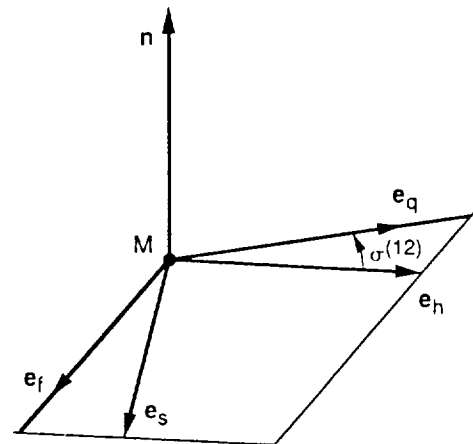


Figure 1.—Unit vectors of principal directions.

where $\mathbf{v}_s^{(1)} = \mathbf{v}_r^{(1)} \cdot \mathbf{e}_s$ and $\mathbf{v}_q = \mathbf{v}_r^{(1)} \cdot \mathbf{e}_q$. These equations relate the relative velocity $\mathbf{v}_r^{(1)}$ of the contact point in its motion over surface Σ_1 with the principal curvatures and directions of the contacting surfaces, the transfer components of the velocities, and the derivative of the gear ratio. It has been shown [4] that the ranks of the augmented matrix for system (1) are 1 and 2 for line and point contact of surfaces, respectively.

(3) Generally, the function of transmission errors for misaligned gear drives is a piecewise, almost linear one (Fig. 2). The approach to be described herein involves a predesigned piecewise function of a parabolic type (Fig. 3) that absorbs the linear function and reduces

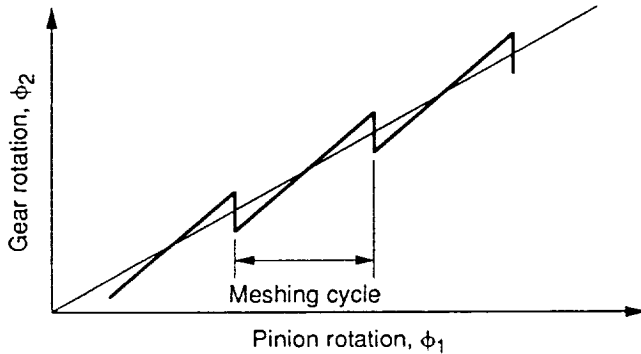


Figure 2.—Piecewise linear function of transmission errors.

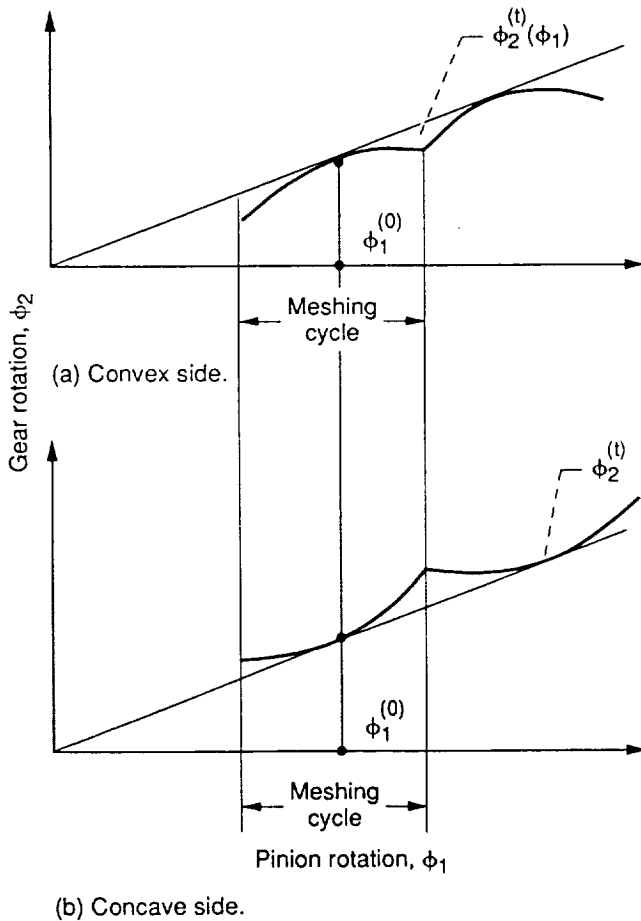


Figure 3.—Parabolic type of transmission errors.

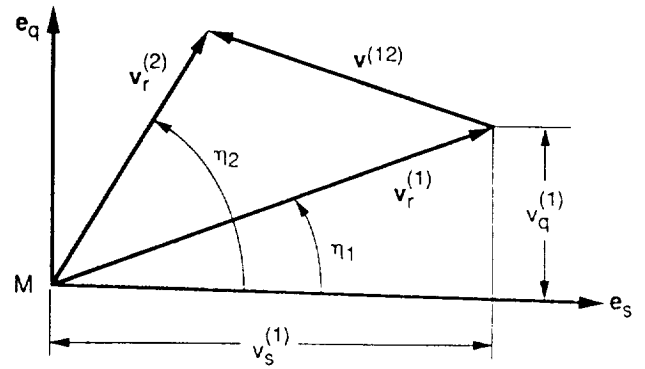


Figure 4.—Tangents to contact paths.

the acceleration jump at the point where the load is transferred between gear tooth pairs.

(4) The relation between velocities $\mathbf{v}_r^{(1)}$, $\mathbf{v}_r^{(2)}$ and the sliding velocity $\mathbf{v}^{(12)}$ at the mean contact point M (Fig. 4) allows one to determine the angle between the tangents to the contact paths and to choose the desired direction of such a tangent on the gear tooth surface.

(5) The desired length of the major axis of the contact ellipse can be obtained by controlling the principal curvatures of the contacting surfaces. The elastic approach, or the deflection of the contacting surfaces under load, is considered to be known.

The results of the analysis that is performed are machine-tool settings. Only small corrections of these settings are required to improve the global meshing and contact conditions that are investigated by tooth contact analysis.

TOOTH CONTACT ANALYSIS

Tooth contact analysis (TCA) permits the simulation of the meshing and contact of gear tooth surfaces that are in continuous tangency. TCA can also simulate the influence of gear misalignment. The outputs from the TCA computer program are the transmission errors and a set of instantaneous contact ellipses that represent the bearing contact pattern along the tooth surface.

The TCA computer program is based on the following equations:

$$\mathbf{r}_f^{(1)}(u_1, \theta_1, \phi_1) = \mathbf{r}_f^{(2)}(u_2, \theta_2, \phi_2, \Delta \mathbf{q}_i) \quad (2)$$

$$\mathbf{N}_f^{(1)}(u_1, \theta_1, \phi_1) = \lambda \mathbf{N}_f^{(2)}(u_2, \theta_2, \phi_2, \Delta \mathbf{q}_i) \quad (3)$$

Here (u_k, θ_k) ($k = 1, 2$) are the Gaussian coordinates of the tooth surface Σ_k ; ϕ_k is the rotation angle of gear k ; and vectors $\Delta \mathbf{q}_i$ represent the gear misalignment with respect to the pinion. The tangency of the gear tooth surfaces is considered in coordinate system S_f , which is rigidly connected to the gear housing. Vector equations (2) and (3) provide that the gear tooth surfaces have a common point and that the normals to the tooth surface at their common point are collinear (the surfaces are in tangency). Vector equations (2) and (3) provide a system of five independent nonlinear equations with six unknowns:

$$f_i(u_1, \theta_1, \phi_1, u_2, \theta_2, \phi_2) = 0 \quad (i = 1, 6) \quad (4)$$

Consider that ϕ_1 is the input parameter. The gear tooth surfaces are in point tangency at every instant if the Jacobian

$$\frac{D(f_1, f_2, f_3, f_4, f_5)}{D(u_1, \theta_1, u_2, \theta_2, \phi_2)}$$

differs from zero.

Equations (2) and (3) are satisfied at the mean point of tangency by the local synthesis. The investigation of global meshing and contact conditions is based on continuing to solve equations (2) and (3) over the active gear surface profile. Only small corrections of the developed machine-tool settings are required owing to proper development of local synthesis.

NUMERICAL EXAMPLE

The following numerical example illustrates the effectiveness of the new approach. The input blank data are given in Table I, the gear cutter specifications are given in Table II, and the gear and pinion machine-tool settings determined by the analysis are given in Tables III and IV. The output of the TCA computer program - the transmission errors and the tooth contact patterns - are shown in Figs. 5 to 7 for three positions of the bearing contact center: (1) located at the mean point (Fig. 5), (2) displaced to the toe (Fig. 6), and (3) displaced to the heel (Fig. 7). The results of the TCA program demonstrate that the developed machine-tool settings provide a low level of transmission errors (6 to 8 arc sec) and a stable tooth contact pattern. Similar results have been obtained for the gear concave side.

TABLE I. - BLANK DATA

	Pinion	Gear
Number of teeth	11	41
Pressure angle, deg	20	----
Shaft angle, deg	90	----
Mean spiral angle, deg	35	----
Hand of spiral	LH	RH
Outer cone distance, mm	----	90.07
Face width, mm	----	27.03
Whole depth, mm	8.11	8.11
Clearance, mm	0.81	0.81
Addendum, mm	5.24	2.061
Dedendum, mm	2.87	6.05
Pitch angle, deg	15°1'	74°59'
Root angle, deg	13°20'	70°39'
Face angle, deg	19°21'	76°40'

TABLE II. - GEAR CUTTER SPECIFICATIONS

Blade angle, deg	20
Cutter diameter, mm	152.40
Point width, mm	2.79

TABLE III. - GEAR MACHINE-TOOL SETTINGS

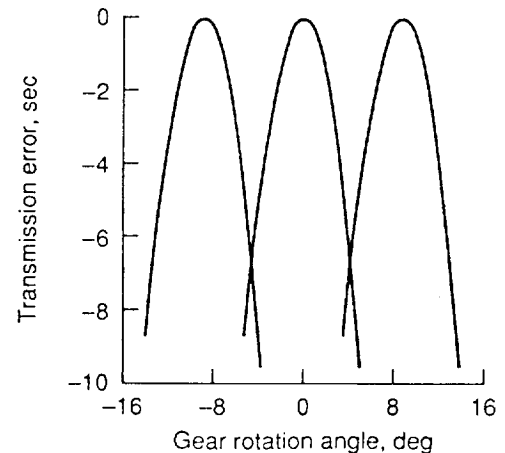
Radial setting, s, mm	70.43577
Cradle angle, q, deg	62.3981
Machine center to back, X_G , mm	0
Sliding base, X_B , mm	0
Ratio of roll, R_a	1.032397
Blank offset, E_m , mm	0
Machine root angle, γ_m , deg	70.65

TABLE IV. - PINION MACHINE-TOOL SETTINGS

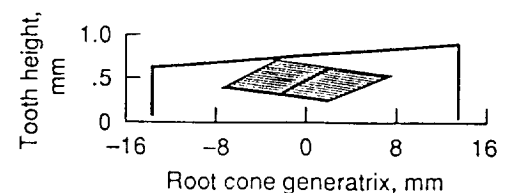
	Convex	Concave
Cutter blade angle, deg	21.5	18.5
Cutter point radius, mm	80.4876	71.7222
Radial setting, s, mm	71.55166	69.04316
Cradle angle, q, deg	59.4638	64.0624
Machine center to back, X_G , mm	1.08497	-1.58960
Sliding base, X_B , mm	-0.25021	0.36659
Ratio of roll, R_a	3.898097	3.788604
Blank offset, E_m , mm	^a -2.56862	^b 2.19033
Machine root angle, γ_m , deg	13.3333	13.3333

^aUp.

^bDown.

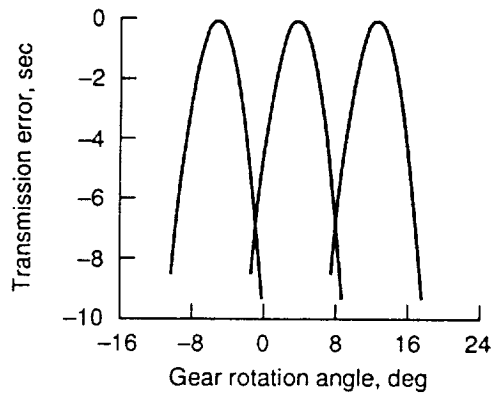


(a) Transmission error in meshing period.

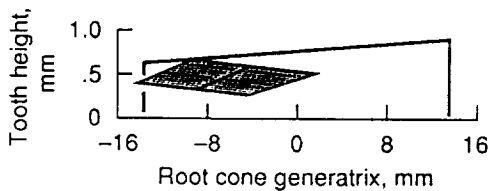


(b) Tooth contact pattern ($V/H = 0/0$).

Figure 5.—Contact pattern and transmission errors at gear mean position of convex side.

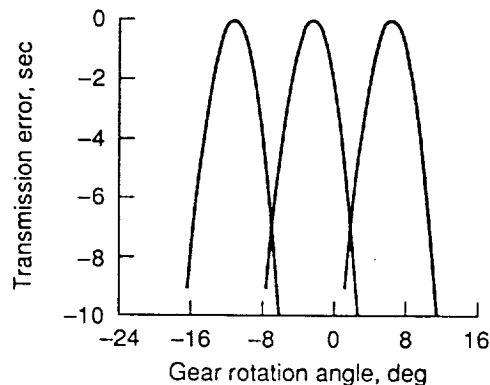


(a) Transmission error in meshing period.

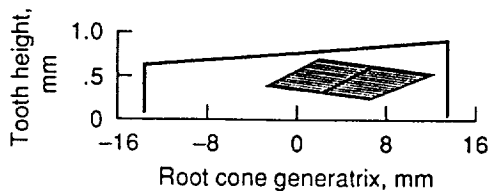


(b) Tooth contact pattern ($V/H = 9/0$).

Figure 6.—Contact pattern and transmission errors at gear toe position of convex side.



(a) Transmission error in meshing period.



(b) Tooth contact pattern ($V/H = -8/3$).

Figure 7.—Contact pattern and transmission errors at gear heel position of convex side.

CONCLUDING REMARKS

An effective method for synthesizing face-milled spiral bevel gears has been developed. Two stages of synthesis have been presented: (1) the local synthesis, and (2) the global analysis and synthesis, which are based on numerical simulation of meshing and contact. The new approach provides a low level of transmission errors and a stable bearing contact without either tilt or modified roll of the head cutter.

REFERENCES

1. Gleason Works: Understanding Tooth Contact Analysis. Rochester, NY 14692, Publication No. SD3139, 1981.
2. Litvin, F.L., and Gutman, Y.: Methods of Synthesis and Analysis for Hypoid Gear Drives of "Formate" and "Helixform," Parts 1-3. Journal of Mechanical Design, Vol. 103, No. 1, Jan. 1981, pp. 83-113.
3. Litvin, F.L., et al.: Topology of Modified Helical Gears, Surface Topology, Vol. 2, No. 1, Mar. 1989, pp. 41-59. (Also, NASA TM-102134.)
4. Litvin, F.L.: Theory of Gearing, NASA RP-1212 (AVSCOM TR-88-C-035), 1989.
5. Litvin, F.L., and Zhang, Y.: Local Synthesis and Tooth Contact Analysis of Face-Milled Spiral Bevel Gears, NASA CR-4342 (AVSCOM TR-90-C-028), 1991.

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